Sr. No. 8170-A

M.Sc. (Maths), First Semester Examination, Dec 2016
Complex Analysis – I

Time: 3 Hours — Max. Marks: 80

Note: Attempt any five questions in all selecting one question from each unit. Unit V is compulsory. All questions carry equal marks.

Unit – I

1. (a) (i) Find the radius of convergence of the series $\frac{z}{2}+\frac{1.3}{2.5}z^2+\frac{1.3.5}{2.5.8}z^3+\cdots$

(ii) Find the domain of convergence of the series $\sum_{n=0}^{\infty} n^2\left(\frac{z^2+1}{1+i}\right)^n$

(b) State and prove Cauchy Hadmaid theorem.

2. (a) Let $f(z)$ be continuous on the regular are L whose equation is $z(t)=x(t)+iy(t), t_0\le t\le T$. Prove that f(z) is intergrable along L and that $\int_L f(z)dz=\int_{t_0}^{T} F(t)[x(t)+ij(t)]dt$ where F(t) denotes the value of f(z) at the point of L corresponding to the parametric value t.

(b) State and prove Cauchy theorem for multiply counted regions.

Unit – II

3. (a) State and prove Cauchy integral formula.

(b) If f(z) is continuous in a region D and if the integral $\int_c f(z)dz$ taken around any closed contom in D vanishes, then f(z) is analytic in D.

4. (a) Show that bounded entire function is identically equal to a constant.

(b) State and prove Taylor's theorem.

Unit – III

5. (a) State and prove Maximum Modulus principle.

(b) Expand $f(z)=\frac{1}{(z+1)(z+3)}$ in a Laurent's series valid for the regions

(i) |z|<1 (ii) 0<|z+1|<2 (iii) 1<|z|<3

6. (a) Let f(z) be analytic inside and on a simple closed contom c except for a finite number of poles inside c, and f(z)≠0 then prove that $\frac{1}{2\pi i}\int_c \frac{f'(z)}{f(z)}dz=N-P$ where N and P repectively the total number of zeros and poles of f(z) inside c.

(b) State and prove inverse function theorem.

Unit – IV

7. (a) State and prove Cauchy Residue theorem.

(b) Using the method of contom integration, prove that $\int_0^{\infty}\frac{x^b}{1+x^2}dx=\frac{\pi}{2}Sec\frac{\pi b}{2}\,(-1<b<1)$

8. (a) State and prove the condition for the mapping w=f(z) to be conformal.

P.T.O.

(b) Find the bilinear transformation which maps the circle |z|=1 into the circle |z-1|=1 and maps w=0, w=1 into $z=\frac{1}{2}$, z=0 respectively.

Unit – V

9. This question is compulsory: (2 marks each)

a) Find the radii of convergence of the power series $\sum\left(1+\frac{1}{n}\right)n^2z^n$.

b) Discuss the transformation $2z = us + \frac{1}{us}$

c) Find the conjugate harmonic of $x^3 - 3xy^2 - 5y$

d) What do you mean by branches of a many valued function?

e) Find the residue at each pole of $f(z) = \frac{2z-1}{(z-1)^4(z+3)^2}$

f) State Gauss Hean value theorem

g) Define simply and multiply connected region

h) State Laurent's theorem

R-11 Reappear

Sr. No. 8169-A

M.Sc. (Maths), First Semester Examination, Dec 2016

Topology

Time: 3 Hours Max. Marks: 80

Note: Attempt any five questions in all selecting one question from each unit. Unit V is compulsory. All questions carry equal marks.

Unit – I

1. (a) Show that for any set E in a topological space c(E)=E υ d(E) 8
   (b) Characterize topology in terms of Kuratowski operator. 8
2. (a) Show that a family β of sets is a basis for a topological space for the set $X = U\{B: B \in \beta\}$ iff for every $B_1, B_2 \in \beta$ and every $x \in B_1 \cap B_2$, there exists a $B \in \beta$ such that $x \in \beta \underline{c} B_1 \cap B_2$ 8
   (b) Give an example to show that Lindelof is not hereditary. 8

Unit – II

3. (a) Let (x,T) and (X^{ℓ}, T^{ℓ}) be two topological spaces A one to one mapping f of X onto X^{ℓ} is homeomorphism if $f(c(E)) = c(f(E))$ for every E$\underline{c}$X. 8
   (b) Show that the product topology is the smallest topology w.r.t. which projection maps are continuous. 8
4. (a) Prove the in a T_1 space, a point x is limit point of a set E if and only if every open set containing x contains an infinite number of distinct points of E. 8
   (b) Show that $\underset{\lambda}{\pi} X_\lambda$ is Hausdroff if and only if each space X_λ is Hausdroff. 8

Unit – III

5. (a) State and prove Embedding Lemma. 10
   (b) Show that a topological space is normal if and only if for any closed set F and an open set G containing F, then there exists an open set G^{ℓ}such that $F \underline{c} G^{\ell}, G^{\ell} \underline{c} G$. 6
6. (a) Show that an ultrafilter converges to a point if and only if that point is a cluster point of it. 8
   (b) Show that the intersection of any non empty family of filters on a non empty set is a filter on X. Is union of two filters also a filter? 8

Unit – IV

7. (a) Show that a toplogical space is compact iff any family of closed sets having finite intersection property has non empty intersection. 8
   (b) Show that compactness is invariant under continuous mappings. 8
8. (a) State and prove Stone-tech compactification theorem. 10
   (b) How is filter convergence related to 6
   i) compactness ii) Product space

Unit – V

9. This question is compulsory: (2 marks each)
   a) Define topological property with example.
   b) Define first and second countable space.
   c) Give an example of a mapping from topological space to another which is open but not continuous.
   d) Define quotient topology.
   e) State ultra filter principle.
   f) Give an example of a normal space which is not completely normal.
   g) State Tychonoff product theorem.
   h) Is power set of a set is filter. Discuss.

Sr. No. 8024

M.Sc. (Maths), First Semester Examination, Dec 2016

Programming in C

Time: 3 Hours Max. Marks: 60

Note: Attempt any five questions in all selecting one question from each unit. Unit V is compulsory. All questions carry equal marks.

Unit – I

1. (a) Explain in detail the structure of C program. 6
   (b) What do you mean by programming language? What are the different programming languages? Explain. 6
2. (a) Explain difference between '=' and '==' operator. Explain with the help of example. 4
   (b) Write a short note on precedence & order of evaluation. 6
   (c) What are logical operators? 2

Unit – II

3. (a) What do you mean by Array? How to store an array in the memory? Explain. 6
   (b) What are Pointers? Explain Pointer arithmetic in detail. 6
4. (a) How to access array elements using pointer? Explain with the help of example. 6
   (b) How to pass array as function arguments? 6

Unit – III

5. Write short note on the following: 4x3=12
   (i) Scope (ii) Global variables (iii) Dynamic Memory Allocation
   (iv) The Register Specifier
6. (a) What are structures and unions? How they are different from each other? 6
   (b) What do you mean by argument conversion? Explain both type of conversions in detail. 6

Unit – IV

7. (a) What do you mean by preprocessors? How do they work in a program? Explain. 6
   (b) What do you mean by live control? Explain. 6
8. (a) How to open and close a file? Explain with the help of example. 6
   (b) What do you mean by unbuffered I/O? Explain. 6

Unit – V

9. Attempt any six question from the following: (2 marks each)
   a) What is an expression? How is an expression is different from variable?
   b) What are Primary data types used in C?
   c) Explain for loop.
   d) What are the rules to declare one dimensional array?
   e) What is function?
   f) What are local variables?
   g) Why enum declaration is used?
   h) What do yu mean by error handling?

Sr. No. 8022

M.Sc. (Maths), First Semester Examination, Dec 2016

Topology – I

Time: 3 Hours | Max. Marks: 80

Note: Attempt any five questions in all selecting one question from each unit. Unit V is compulsory. All questions carry equal marks.

Unit – I

1. (a) State and prove confinite topology on a non-empty set X. 6

(b) Prove that in a top space (X,j), $\overline{A} = A \cup d(A) \forall A \subseteq X$. 10

2. (a) Prove that a family β of sets is a base for a topology for the set $X = U\{B: Bt\beta\}$ iff for every $B_1, B_2 \in \beta$ and every $x \in B_1 \cap B_2, \exists\ a\ B \in \beta$ s.t. $x \in \beta \underline{c} B_1 \cap B_2$. 8

(b) Characterize topology in terms of Kuratowsti closure operator. 8

Unit – II

3. (a) If f is a mapping of a top space X into another top space, then prove that f is continuous on X iff $f(c(E))\underline{C}C^{*}f(E)\forall E\underline{C}X$. 8

(b) If a connected set C has a non-empty intersection with both a set E and the complement of E in a top space (X,j), then show that C has a non-empty entersection with the boundary of E. 8

4. (a) Define component of a space. Is componenet of a space necessarily open? 8

(b) Prove that every protection $\pi_\lambda: X \to X_\lambda$ on a product space $X = \underset{\lambda}{\overset{\pi}{}} X_\lambda$ is both open and continuous. 8

Unit – III

5. (a) Show that every second countable space is first countable but converse may not be true. 8

(b) State and prove Lindelof theorem. Also show that converse of Lindelof theorem is not true. 8

6. (a) Define T_0 and T_1 –spaces and show that a top space is T_1 iff every subset consisting of exactly one point is closed. 8

(b) Show that the property of being a T_2-space is a topological property. 8

Unit – IV

7. (a) Define a compact set. Show that an infinite set with cocountable topology is not compact. 8

(b) Show that a top space is compact iff any family of closed sets having FIP has a non-empty intersection. 8

8. (a) Prove that every sequentially compact top space is countably compact. 8

(b) Let (X*, j*) be a one point compactification of a non-compact top space (X,j), then prove that (X*,j*) is a Hausdorff space iff (X,j) is locally compact. 8

Unit – V

9.

a) Show that the union of two or more topologies need not be a topology.
b) Prove the $(A \cap B)^0 = A^0 \cap B^0$
c) Define relative topology.
d) Define open and closed mappings.
e) Show that every closed subset of a compact space is compact.
f) Show that cofinite topology is a T_0-space.
g) Prove that every indiscrete space is locally compact.
h) Define hereditary and topological properties.

Sr. No. 8023

M.Sc. (Maths), First Semester Examination, Dec 2016

Differential Equation – I

Time: 3 Hours Max. Marks: 80

Note: Attempt five questions in all, selecting one question from each unit and the compulsory question.

Q.1 Compulsory question: 2x8=16

i) Define a uniformaly Lipschitz continuous function defined on RxR^n.

ii) Reduce the IVP $y''(t) - 2ty'(+) - 3y(t) = 0, y(0) = 1, y'(0) = 0,$ to integral equation.

iii) State Kenser's Theorem.

iv) Explain the concept of equicontinuous family of functions.

v) If a fundamental matrix $\Phi(t)$ for $x' = A_x$ exists for $0 \le t \le t_1$, prove that $\Phi^{-1}(t)$ exists and is continuously differentiable for $0 \le t \le t_1$.

vi) Represent the system of equations $x_,'' - 3x_,' + 2x_, + x'_2 - x_2 = 0$

$x'_, - 2x_, + x'_2 + x_2 = 0$, in matrix form.

vii) Obtain Picard's approximations of the differential equation $x' = t + x$, where $x_0(t) = e^t$.

viii) Define the concept of minimal solutions with a suitable example.

UNIT – I

Q.2 (a) State and prove Couchy-Euler construction theorem for an approximate solution of an IVP. 10

(b) State and prove Ascoli Lemma. 8

Q.3 State and prove Picard-Lindelof theorem. 16

UNIT – II

Q.4 (a) If Φ is a fundamental matrix of the system $x' = Ax$ and c is a constant non-singular matrix, then prove that Φc is a fundamental matrix. Also prove that every fundamental matrix of the system is of this type for some non-singular matrix. 8

(b) Define Adjoint system to a linear homogeneous system. State and prove the relationship between fundamental matrices of the two systems. 8

Q.5 (a) Explain the procedure to reduce the order of homogeneous sytem. 8

(b) State and prove the variance of constant formula for non-homogeneous linear system. 8

UNIT - III

Q.6 (a) State and prove Abel's Identity for an n^{th} order homogeneous linear differential equation. 8

(b) State and prove the necessary and sufficient condition for n solutions of homogeneous linear differential equation of order n to be linearly independent. 8

Q.7 (a) If $f_1, f_2, \dots, f_n$ be a set of n functions, each of which has a continuous n-th derivative on $a \le t \le b$ such that wronskian of these functions is non-zero for all t on $a \le t \le b$. Find out a unique normalized homogeneous linear differential equation of order n, which has $f_1, f_2, \dots, f_n$ as a fundamental set on $a \le t \le b$. 8

(b) State and prove the Lagrange's Identity associated with the nth order linear differential operator. 8

UNIT - IV

Q.8 (a) State and prove the basic theorem concerning the dependence of solution of an IVP on initial conditions. 8

(b) Prove Kamke's General Uniqueness theorem. 8

Q.9 Write a note on uniqueness of solution. State and prove the following Uniqueness theorem. 16

i) Osgood's theorem

ii) Nagumo's theorem

Sr. No. 8167-A

M.Sc. (Maths), First Semester Examination, Dec 2016

Advanced Abstract

Time: 3 Hours

Max. Marks: 80

Note: Attempt any five questions in all selecting one question from each unit. Unit V is compulsory. All questions carry equal marks.

Unit – I

1. (a) State and prove Cauchy theorem for finite Abelian groups. 8

(b) State and prove Sylow's Third Theorem. 8

2. State and prove Jordan Holder Theorem for finite groups. 16

Unit – II

3. (a) Prove that finite extension of a finite extension is also a finite extension. 8

(b) Let charF=0 and a, b are algebraic over F. Show that F(a,b)=F(c) for some c∈ F(a,b). 8

4. (a) Let f(x)∈F[x] be any polynomial of degree n. Prove that there exists an extension K of F which contains all roots of f(x) and [K:F]≤ $\lfloor^{n}$ 8

(b) Prove that a field F is finite iff its multiplicative group F*=F-{0} is cyclic. 8

Unit – III

5. (a) Prove that every algebraic extension of a finite field F of characteristic p is separable extension. 8

(b) Find the Galois group of the polynomial $x^4 - 5x^2 + 6$ over rationals. 8

6. State and prove the fundamental theorem of Galois theory. 16

Unit – IV

7. (a) Show that a general polynomial of degree 5 is not solvable by radicals. 8

(b) Prove that a group of order p^n(p-prime, n≥1) is always solvable. 8

8. Prove that $\phi_n(x) = \underset{w}{\pi}(x - w), w$ is primitive n^{th} root in C, is an irreducible polynomial of degree $\phi(n)$ *in* $Z[x]$. 16

Unit – V

9. 16

a) Define inner automorphism of a group G

b) State Zassenhaus Lemma

c) Define conjugate elements

d) Define a normal extension

e) Define Galois extensions

f) State theorem of primitive element

g) Define cyclic extension

h) Define solvable group

Fresh

Sr. No. 8021

M.Sc. (Maths), First Semester Examination, Dec 2016

Real Analysis – I

Time: 3 Hours Max. Marks: 80

Note: Attempt any five questions in all selecting one question from each unit. Unit V is compulsory. All questions carry equal marks.

Unit – I

1. (a) If $f\epsilon R(\alpha)$ on [a,b], then show that $f\epsilon R(\alpha)$ on [a,c] and [c,b] where $c\epsilon[a,b]$ and $\int_a^b fd\alpha = \int_a^c fd\alpha + \int_c^b fd\alpha$ 8

   (b) State and prove second mean value theorem for Riemann Stieltjes integral. 8

2. (a) If f maps (a,b) into R^k and if $f\epsilon R(\alpha)$ for some monotonically increasing function α on (a,b), then $|f|\epsilon R(\alpha)$ and $|\int_a^b fd\alpha| \leq \int_a^b |f|d\alpha$ 8

   (b) Show that $f\epsilon R(\alpha)$ on [a,b] iff for every $\varepsilon > 0$, there exists a partition P of [a,b] such that $U(P, f, \alpha) - L(P, f, \alpha) < \varepsilon$. 8

Unit – II

3. (a) Define an algebra of sets. Show that collection of all measurable sets is an algebra. 8

   (b) Consturt a non-measurable set on (0,1). 8

4. (a) Show that the interval (a, ∞) is measurable. 8

   (b) Let $\{E_i\}$ be an infinite decreasing sequence of measurable sets. Let $m(E_i) < \infty$ for atleast one $i \in N$. Then, prove that $m(\bigcap_{i=1}^{\infty} E_i) = \lim_{n\to\infty} m(E_n)$. 8

Unit – III

5. (a) Show that characteristic function x_A is measurable iff A is measurable. 8

   (b) Show that if f is a function defined on o measurable set E then f is measurable iff for any open set G is R then $f^{-1}(G)$ is also measurable. 8

6. (a) State and prove F. Riesz theorem for convergence in measure. 8

   (b) State and prove Egoroff Theorem. 8

Unit – IV

7. (a) State and prove Bounded convergence theorem. 8

   (b) Write shortcomings and advantages of Lebesgue integral over Riemann integral. 8

8. (a) State and prove Monotone convergence theorem. 8

   (b) Show that the function $f: (0, \infty) \to R$ defined as 8

$$f(x) = \begin{cases} \frac{sinx}{x} & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$$

P.T.O.

Unit - V

9. 16

a) Evaluate $\int_0^3 [x] dx^2$

b) State first mean value theorem for Riemann Stieljes Integral.

c) Define set function.

d) Define σ −algebra.

e) Define measurable function.

f) State Littlewood's 2nd principle.

g) State Labesgue dominated convergence theorem.

h) Define Lebergue integral for simple function